April 11, 1867.

Lieut.-General SABINE, President, in the Chair.

The following communications were read :—

I. "A new fact relating to Binocular Vision." By A. CLAUDET, F.R.S. Received March 20, 1867.

The persistence of the impression made by light on the retina is demonstrated by many experiments; but one of the most convincing, which is also very easy to try, is that which is known under the name of the thaumatrope.

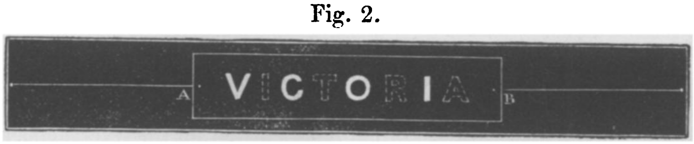
Let us write the letters composing a word of eight letters, say "Victoria," on the two sides of a small card, in such a manner that one surface shall contain the 1st, 3rd, 5th, and 7th letters, and the other surface the 2nd, 4th, 6th, and 8th, with a space between them sufficient to complete the word on each surface, which blank spaces are in fact to appear filled up during the experiment by an artificial means to be explained.

Fig. 1 shows the arrangement on the two sides of the card (section view).

Fig. 1.

Fig. 2 shows the plan of the card. The white letters are those written on one surface, and the dotted lines those written on the other.

Fig. 2.



Now by means of two strings fixed on the two sides A, B the card may be made to revolve on its axis by turning the string between the thumb and finger of each hand. By this means a very rapid motion may be communicated to the card, and while it is revolving both surfaces are alternately seen in quick succession, and the perception of the two is so simultaneous that the two sets of letters appear as one, and the whole word is read as distinctly as if it were written on one surface only.

This is easily explained. It is known that the persisting action of light on the retina has a duration of about one-eighth of a second; so that if the card makes at least eight revolutions in a second (it may make considerably more), before one impression has vanished another produces its effect on the next part of the retina, in such a way that they are intermixed and simultaneously visible, producing an uninterrupted sensation.

The means by which this illusion is produced has been called the "thaumatrope," from two Greek words meaning "wonder" and "turn." It is difficult to trace the history of this discovery; but it is certain that it has been the result of a very old, simple, and well-known experiment.

From time immemorial schoolboys have amused themselves by holding a coin between two pins and making it revolve rapidly by blowing upon it, when to their surprise the coin showed the head mixed with the device on the other side. I have been told that as Sir John Herschel was one day making this experiment to amuse his children, in the presence of the late Dr. Paris, this gentleman was struck with the idea that if, instead of a coin, a white card was employed on each side of which one part of a design was properly arranged, the two might complete the subject during the revolution. Accordingly he made the experiment, which succeeded perfectly well. If the story is true, certainly Dr. Paris may be regarded as the inventor of the thaumatrope, which he has so well and so fully elucidated in his very interesting and instructive work entitled 'Philosophy in Sport made Science in earnest.'

This philosophical toy may be employed to show another effect of the persistence of the retinal image. If complementary colours are fixed on the two opposite sides of the card, they will become superposed during the revolution, and white light will be the result. By the same means, other curious effects of the mixture of various colours might be tried.

All these experiments present no difference, whether they are made looking with the two eyes or only with a single eye: the effect is the same in both cases. Therefore the illusion is equally monocular and binocular.

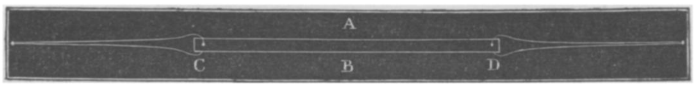
But I was not a little surprised to find that the thaumatrope is capable of producing another phenomenon, elucidating very forcibly the principles by which binocular vision is the only real and effective means of showing the distances of objects, which are determined by the degree of the angle of convergence of the optic axes and by one of its corollaries, the sensation of double images for all the points which are not exactly on the plane of vision.

The thaumatrope is capable of showing that binocular vision can detect to a degree hardly conceivable the most minute difference in the distances of objects, such as the distance between the planes of the two surfaces of a card, which distance is nothing more than the thickness of the card. Therefore, supposing that the thickness of the card is $\frac{1}{80}$ of an inch and the distance from the eyes 15 in, there is not a difference greater than the $\frac{1}{1200}$ part of the whole distance from the eyes to the two planes of the card; and still the difference of the degree of convergence for two planes so near each other is sufficient to excite the action of binocular vision, and by it to enable us to detect that infinitesimal difference in their distances. But that such an effect of binocular vision could possibly be displayed while looking at two planes so nearly intermixed as the surfaces of a card revolving upon its axis with such a wonderful velocity is the very extraordinary pheno-

menon I have discovered, and which I am about to describe and endeavour to explain.

If the thickness of the card is A B, fig. 3, and if the two ends of each

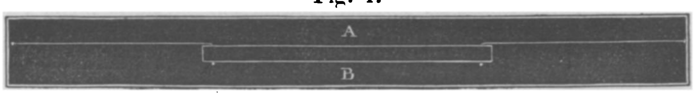
Fig. 3.



string, passing through the holes C and D in the card, are brought together and turned between the thumb and finger, the card will whirl exactly on its axis, and during the revolution the two surfaces A and B will be at the same distance from the eyes.

But if the two strings are drawn so that one of their knots is as in fig. 4,

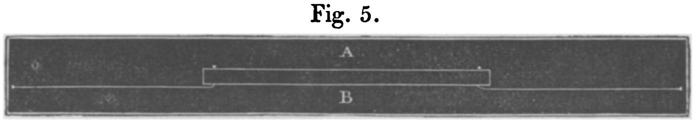
Fig. 4.



the surface B will revolve round the plane of the surface A corresponding with the axis of the string, and, during the revolution, every time that it is made visible to the eyes it will appear as if it were nearer than the surface A.

By reversing the position of the knots, as in fig. 5, instead of the surface

Fig. 5.



B revolving round the plane of A, it will be A that will revolve round the plane of B.

These three different positions of the strings will produce three different effects.

In the position of fig. 3 the effect will be normal; that is to say, the two surfaces coming alternately at the same distance, we shall see the whole word as if the letters were on the same surface.

In the position of figs. 4 and 5 we have a very strange illusion. One half of the letters composing the word will appear before or behind the other half, according to the surface upon which they are written and the position of the knots upon that or the other surface.

In fig. 4 the letters written on the surface B will appear before the letters on the surface A; and in fig. 5 the letters on the surface A will appear before the letters on the surface B.

The cause of the anomaly resulting from the two different experiments is entirely and positively due to a sensation of binocular vision; and we may

easily satisfy ourselves that it is so; for, looking with a single eye in both cases, all the letters appear on the same plane, notwithstanding the different distances of the two surfaces given by the position of the knots: and we may add another convincing proof, which is that the pseudoscope inverts the distances of the surfaces.

At first it is rather difficult to understand how the phenomenon can take place; for as the perception of the two surfaces is simultaneous, how is it possible that during such a rapid revolution the optic axes can be made to converge alternately on each surface while it is passing so quickly, and that they should be made suddenly to converge on the other in its turn?

However, there cannot be any question that in reality the phenomenon takes place, and that it is decidedly an effect of binocular vision; therefore it only remains to be explained how it can be produced. In endeavouring to arrive at the true cause of the phenomenon, we shall have to bring to mind various physiological sensations which concur in producing the effect. One is the effort we make to obtain distinct vision, and the other the effort we make to obtain single vision. These two efforts act in unison; for it is impossible not to admit that the two muscular processes by which both the angle of convergence is directed to the object and the focus of the eyes is adapted to its distance, for the double purpose of having at once single and distinct vision of every object, are two actions necessarily simultaneous and inseparably connected. They are therefore both, each in its way, criteria of the distances of objects; but they give rise to certain indirect and additional criteria for other distances, in two ways: one, the most important, is the double images of the objects situated before and behind the point of convergence; and the other, but only in a subsidiary way, the degree of confusion of the objects situated before and behind the point of convergence and which are not in focus.

The comparison of two points, one of which is in focus and well defined, and the other out of focus and confused, helps considerably in forming a judgment that they are on different planes. But in a question of binocular vision, perhaps we ought not strictly to take into account this last criterion, which belongs equally to monocular and binocular vision; and if we allude to it, it is only because, although it does not produce the real stereoscopic effect, still it contributes to give that sort of illusion of relief which by various means may be evinced by monocular vision. Therefore it is particularly the sensation of double images, the degree of their separation, and their respective positions either outside or inside from the centres of the two retinæ, which indicate more powerfully the exact distance of the object from the point of single vision either before or behind.

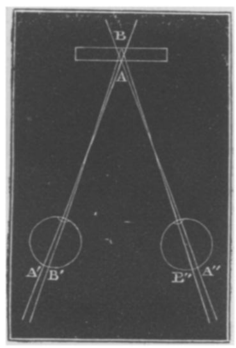
When we look fixedly on a point of one surface of the revolving card, that point appears single, and we see at the same time another point on the other surface which appears double, although we hardly feel that we notice its doubleness; and from the position or distribution of the double images, either on the right or on the left of the central point, we have at

the same glance the perception of the respective distances. Therefore, to judge of the distances of certain objects in the direction of the line of vision, we are not absolutely obliged to alter constantly the angle of convergence. This is proved by our perception of the two distances of the surfaces of the card while it is revolving; for it would be impossible that we should alter the angle of convergence to adapt it alternately to the two surfaces while they are turning so rapidly.

The same angle of convergence kept on one or the other surface is no impediment to our seeing both in a sufficiently distinct manner.

The whole phenomenon may be better understood by the illustration given in fig. 6.

Fig 6.



When we converge the optic axes on B, this point, being represented on the centre of both retinæ at B′ B″, is single, but A being nearer is represented on the left of the centre of the left retina at A′, and on the right of the centre of the right retina at A″; therefore it appears double.

For the same reason, converging on A, this point is single, but B is double, with this difference—that one image is on the right of the left retina, and the other on the left of the right retina; so that the double images of nearer objects situated at A are represented outside the centres of the two retinæ, and those of further objects situated at B are represented inside the centres of the retinæ, and each of these two different sensations brings to our mind the perception of the distance which has produced it. During the revolution of the card we may adapt the convergence either to one or to the other surface and keep it so; but in every case the letters on that surface will appear single and a little better defined; and this, with the sensation of double images of the letters on the other surface, will be an indication of their respective distances.

As I am not aware that the illusion I have described in this paper has ever been noticed before, it has appeared to me that its publication would excite the interest of all those who look for any new fact capable of illustrating the principles of binocular vision, and showing the wonderful pro-

perty of the angle of convergence, by which the most minute differences in the distances of objects and the slightest relief on their surfaces can be detected, and by which also in the abnormal conversion introduced in its action by the pseudoscope all our sensations are reversed. Therefore the pseudoscope is the great test of the phenomena of binocular vision; for by reversing certain sensations which by constant habit we may hardly notice, it renders them more conspicuous by the comparison of the abnormal state brought out by its action, and proves the theory of binocular vision in the most effective manner.

A truth is never better established than when it can be shown that the same principles are capable of producing contrary effects when they are applied in a contrary way.

Professor Wheatstone, by adding the pseudoscope to the stereoscope, has thus in the most scientific and ingenious manner completed his splendid discovery, and left very little (we might almost venture to say that he has left nothing) for further investigations in the physiology of binocular vision.

II. "On the Calculation of the Numerical Value of Euler's Constant, which Professor Price, of Oxford, calls E." By WILLIAM SHANKS, Esq., Houghton-le-Spring, Durham. Communicated by the Rev. B. Price, F.R.S. Received March 28, 1867.

In the year 1853 Dr. Rutherford, of the Royal Military Academy, Woolwich, sent a paper on the Computation of the value of π to the Royal Society, and the paper was published in the 'Proceedings' of that learned body*. The value of π is there given to 607 decimals, the first 440 being the joint production of Dr. Rutherford and the author of this paper, and the remaining 167 decimals having been calculated by the present writer, for the accuracy of which he alone is responsible. Subsequently, the Astronomer Royal, G. B. Airy, Esq., kindly presented the author's paper on the Calculation of the value of e, the base of Napier's logarithms, to upwards of 200 decimals; the aforesaid paper also contained the Napierian logarithms of 2, 3, and 5, as well as the modulus of the common system, all to upwards of 200 places of decimals. This paper was not, however, published, but deposited in the Archives of the Royal Society; but an abstract, containing the numerical results, was printed in the 'Proceedings'†. In a paper sent by the author to the Astronomer Royal, and forwarded by him to the Royal Society, will, the author believes, be found the reciprocal of the prime number 17389, consisting of a circulating period of no less than 17388 decimals, the largest on record. Some few remarks are also given touching circulates generally, and the easiest modes of obtaining them. The writer now desires to supplement what he then did, by giving the

* Vol. vi. p. 273. † Vol. vi. p. 397.